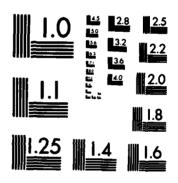
ELECTRIC FIELDS IN EARTH ORBITAL SPACE(U) MCDONNELL DOUGLAS ASTRONAUTICS CO-HB HUNTINGTON BEACH CA W P OLSON ET AL. SEP 84 MDC-H1666 N00014-80-C-0796 F/G 4/1 AD-A146 448 1/1 UNCLASSIFIED NL



'OPY RESOLUTION TEST CHART

AD-A146 448

AND THE RESIDENCE OF THE PARTY OF THE PARTY

ANNUAL REPORT ELECTRIC FIELDS IN EARTH ORBITAL SPACE

SEPTEMBER 1984

MDC-H1666

MCDONNELL DOUGLAS ASTRONAUTICS COMPANY

MCDONNELL DOUGLA

CORPORATION

907 5 1984

24-10-01-120

THE CODE





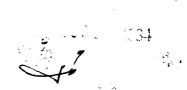
ANNUAL REPORT ELECTRIC FIELDS IN EARTH ORBITAL SPACE

SEPTEMBER 1984

MDC-H1666

Prepared by

W. P. Olson K. A. Pfitzer S. Schneider



MCDONNELL DOUGLAS ASTRONAUTICS COMPANY-HUNTINGTON BEACH
5301 Bolsa Avenue Huntington Beach, California 92647 (714) 896-3311

| SECURITY CLASSIFICATION OF THIS PAGE | | | | | |
|---|--|--|--|--|--|
| | REPORT DOCU | MENTATION | PAGE | | |
| 1a. REPORT SECURITY CLASSIFICATION Unclassified | | 1b. RESTRICTIVE MARKINGS None | | | |
| . SECURITY CLASSIFICATION AUTHORITY | | 3. DISTRIBUTION/AVAILABILITY OF REPORT | | | |
| 2b. DECLASSIFICATION / DOWNGRADING SCHEDULE | | Unlimited | | | |
| 4. PERFORMING ORGANIZATION REPORT NUMBER(S) | | 5. MONITORING ORGANIZATION REPORT NUMBER(S) | | | |
| MDC-H1666 | | | | | |
| 6a. NAME OF PERFORMING ORGANIZATION McDonnell Douglas Astronautics Company | 6b. OFFICE SYMBOL (If applicable) | 7a. NAME OF MONITORING ORGANIZATION | | | |
| 6c. ADDRESS (City, State, and ZIP Code) 5301 Bolsa Avenue Huntington Beach, CA 92647 | | 7b. ADDRESS (City, State, and ZIP Code) | | | |
| 8a. NAME OF FUNDING/SPONSORING ORGANIZATION | 8b. OFFICE SYMBOL (If applicable) | 9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER | | | |
| Office of Naval REsearch | L | 10 5011855 05 1 | THAT DIALS AND ADD | : O.C | · |
| 8c. ADDRESS (City, State, and ZIP Code) Washington, D.C. | | 10. SOURCE OF F PROGRAM ELEMENT NO. | PROJECT NO. | TASK NO. | WORK UNIT ACCESSION NO. |
| washington, b.c. | | W8QA2002 | | 1 | ACCESSION NO. |
| 11. TITLE (Include Security Classification) Electric Fields in Earth Orbita | 1 Space (U) | | | | |
| 12. PERSONAL AUTHOR(S) P. Olson, K. A. PFitzer, S. | Schneider | | | | |
| Annual 13b. TIME C | 5/83 to <u>9/84</u> | 14. DATE OF REPO September | PRT (Year, Month 27, 1984 | , <i>Day</i>) 15 | . PAGE COUNT 48 |
| 16. SUPPLEMENTARY NOTATION Electromagnetic waves Electromagnetic plasmas | magnetosp magnetic electric | fields | | | |
| 17. COSATI CODES | 18. SUBJECT TERMS | | e if necessary ar | nd identify | by block number) |
| FIELD GROUP SUB-GROUP A 2003 | 1 | [, | • | • | • |
| 19. ABSTRACT (Continue on reverse if necessary | <u> </u> | | | | |
| state magnetosphere prior to the formed and maintained simply by It is known, however, that the planetary magnetic field (IPMF) believe is basically incorrect) wave propagation in the interple (solar wind) is magnetized by thigher frequency can propagate the interaction of these disturnagnetospheric response to the electromagnetic disturbances in magnetosphere. Only certain most the magnetopause. We show the solution of the selectromagnetic disturbances in magnetosphere. Only certain most the magnetopause. We show the solution of the selectromagnetic disturbances in magnetosphere. Only certain most the magnetopause. We show the solution of the selectromagnetic disturbances in magnetosphere. Only certain most the magnetopause. We show the solution of the selectromagnetic disturbances in magnetosphere. | the interaction magnetosphere recommended. Instead of the solar magnet through the solar magnet through the solar may be a the interplane odes propagate a chat a southward magnet magnet a southward magnet magnet magnetation and the solar magnetation of the solar magnetation and | n of the solution of the supports dynamic dyna | ar wind wit mically to ve reconnect ponse in te that the iructure. Experience than titatively and their if urther reses the most | th the g changes tion therms of nterpla lectromable at it cause the pr nteract trictio | eomagnetic field. in the inter- eory (which we electromagnetic netary plasma agnetic waves of tenuation. It is s the observed copagation of tion with the ens on the waves |
| Za. NAME OF RESPONSIBLE INDIVIDUAL | | 22b. TELEPHONE (| Include Area Cod | (e) 22c. Ol | FFICE SYMBOL |
| DD FORM 1473, 84 MAR 83 A | PR edition may be used u | ntil exhausted. | SECTION | CI ASSISIC | ATION OF THIS PAGE |

83 APR edition may be used until exhausted. All other editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE

UNCLASSIFIED

19. Abstract (Continued)

ww

C

magnetospheric response in the tail region and a northward IPMF in the polar cusp region. Further work on propagation of these fields within the magnetosphere and on a quantitative examination of their oblique incidence on the magnetopause is suggested. We believe this work will help us to quantitatively understand the magnetosphere's dynamic response to the IPMF and should lead to a quantitative predictive capability for many magnetospheric features.



A-1

TABLE OF CONTENTS

| <u>Section</u> | <u>Title</u> | <u>Page</u> | | | | |
|----------------|---|-------------|--|--|--|--|
| | FOREWORD | 1 | | | | |
| 1 | TECHNICAL DISCUSSION | 3 | | | | |
| | 1.1 Steady State Magnetospheric Response to the the solar wind | .3 | | | | |
| | 1.2 Magnetospheric Response to Solar Wind Variability | 6 | | | | |
| | 1.3 Magnetospheric Response to the Interplanetary Magnetic Field | 7 | | | | |
| | 1.4 Conclusion on Wave Propagation in the Solar Wind | 8 | | | | |
| | 1.5 Reflection and Refraction at the Magnetopause | 9 | | | | |
| | 1.6 Technical Summary | 13 | | | | |
| 2 | FUTURE WORK | 16 | | | | |
| | 2.1 Oblique Incidence | 16 | | | | |
| | 2.2 The Effects of Interplanetary Electromagnetic Disturbances on Magnetospheric Plasmas and Fields | 16 | | | | |
| | 2.3 Influences in the Dayside Cusp Regions | 17 | | | | |
| Appendix A | GENERAL DESCRIPTION OF SOLAR WIND AND INTERPLANETARY MAGNETIC FIELD PROPERTIES | | | | | |
| Appendix B | PROPAGATION OF ELECTROMAGNETIC DISTURBANCES IN THE SOLAR WIND | | | | | |
| | B.1 Propagation in Vacuum | 25 | | | | |
| | B.2 Propagation in the Presence of a Plasma | 25 | | | | |
| | B.2.1 General Equations | 25 | | | | |
| | B.2.2 Propagation in a Plasma With No Background Magnetic Field | 35 | | | | |
| | B.2.3 Propagation in a Plasma With an Imbedded Magnetic Field | 37 | | | | |
| | B.2.3.1 Propagation Parallel to \underline{B} | 38 | | | | |
| | B.2.3.2 Propagation Perpendicular to $\underline{\mathbf{B}}$ | 43 | | | | |
| | References and Recent Publications | 46 | | | | |

FOREWORD

A proportion of the material provided in this report was provided to your office previously in the form of a proposal submitted in May 1984. Work on technical progress has been been excerpted from that proposal and included in his report. In addition, this report contains

A more detailed mathematical description of very low frequency electromagnetic wave propagation in the solar wind plasma (see Appendix B).

A discussion on why wave propagation in a plasma is not allowed when the electric portion of the disturbance vector is parallel to the ambient magnetic field (see Appendix B).

An examination of the special effects that may be present during sector crossings (see Section 1.3).

An expanded discussion on the anticipated effects of these interplanetary magnetic disturbance waves on magnetospheric processes (see Section 2.2).

Early in the study of the magnetosphere, its size and shape were quantitatively determined by calculating the balance of pressure between the geomagnetic field energy density and the kinetic energy of the solar wind at the magnetopause. On an individual particle basis, the pressure balance condition suggests that all solar wind electrons and protons (and other positive ions) are deflected specularly off the geomagnetic field at the magnetopause. Thus, a "closed" magnetosphere is formed where no solar wind particles are allowed to penetrate through the magnetopause. Also, in this closed magnetosphere the deflection of solar wind particles is responsible for the formation of a current system at the magnetopause whose magnetic field exactly offsets the geomagnetic field outside of the magnetosphere. In this model, no geomagnetic field lines penetrate the magnetopause.

Although there have been successful attempts at quantitatively explaining several microphysical processes in the magnetosphere (e.g., the decay of the ring current), the pressure balance formalism and associated quantitative determination of magnetopause shape and size remains, twenty years after its postulation, the only example of gross magnetospheric features explained on a physical basis. The observed shape and size of the magnetosphere are in good agreement with these predictions of the pressure balance models. Yet, after years of observations, it is now known that several features persist in the magnetosphere that are not explained by the pressure balance model of the closed magnetosphere. These include the plasma sheet (a region of relatively high plasma density sandwiched between the two lobe regions in the tail), an electrical current system that flows from dawn to dusk across the tail and returns at or just beyond the magnetopause, an electrostatic field directed from dawn to dusk across the tail of the magnetosphere, a "boundary layer" of relatively dense plasma moving typically in the anti-solar direction and situated just inside of the classical magnetopause, and cusped regions in the high latitude dayside magnetosphere in place of the neutral points predicted by the pressure balance models.

Over the past two years, we have attempted to explain on a physical basis the persistence of these large scale magnetospheric features. Generally our work can be described in two parts: first, our attempt to explain the feature itself and its persistence at all times and second, an attempt to quantitatively understand the reasons for the observed variability in each of these features.

Section 1 TECHNICAL DISCUSSION

We believe that we can explain the persistence and average properties of almost all gross magnetospheric features simply by re-examining the pressure balance formalism which assumes, as stated above, the specular reflection of all particles off the geomagnetic field. On an individual particle basis, the pressure balance formalism implies that each particle moves over a portion of a circular path while it is encountering the geomagnetic field so that its angle of incidence to the magnetopause is exactly equal to its angle of reflection (thus the term specular, or mirror-like, reflection). However, it has been known for a long time that other more energetic particles from the sun (e.g., solar cosmic rays) do gain access to the earth's magnetosphere. It is generally argued that the reason for this is that the more energetic particle has such large gyroradii that the assumption of uniform geomagnetic field over the region where the particle initially encounters the geomagnetic field is not valid. Thus, it is possible for solar cosmic rays to sample a large enough portion of the nonuniform geomagnetic field for some of them to gain permanent access to the magnetosphere. This particle entry concept and an accurate model of the geomagnetic field have been used to explain the low latitude cutoffs observed for solar cosmic rays on polar orbiting satellites (Pfitzer, 1979).

1.1 Steady State Magnetospheric Response to the Solar Wind

It is therefore logical to ask the following question. What is the lowest energy charged particle that can gain entry to the magnetosphere because of the gradients in the geomagnetic field that persist at the magnetopause? It is clear that the assumption of specular reflection is only an approximation because of the gradients in the geomagnetic field known to exist at the magnetopause. Both model calculations based on the pressure balance assumption and observed fields are on the order of 75 nT in the sub-solar region, and diminish to only several nTs in the equatorial region of the tail of the magnetosphere. We have calculated the trajectories of tens of

thousands of charged particles as they impact the magnetospheric magnetic field at various points on the magnetopause (in a field model that included an accurate representation of these gradients. We found that some particles with solar wind energies (~ one keV proton and lower) have access to the magnetosphere provided that they impact the given point within an allowed cone of incidence directions. An example of an entry cone is given in Figure 1. It is found generally that the entry cone itself is largest at the magnetic equator and increases monotonically from a zero value at the sub-solar point along the intersection of the magnetic equator with the dawn side of the magnetopause. Thus it is expected that charged particle entry to the magnetosphere occurs most readily near the magnetic equator in the tail of the magnetosphere.

This gradient drift entry (GDE) mechanism possesses another important feature that helps explain several permanent magnetospheric features. That is, only positively charged particles can enter on the dawn half of the magnetopause and only electrons can enter on the dusk half. The noon-midnight meridian intersection with the magnetopause is a symmetry plane over which no entry occurs. (Note that along the noon-midnight meridian the magnetospheric magnetic field does possess a symmetry and there particle interactions with the field can be treated with the specular reflection assumption.) This feature is especially important regarding processes and features in the tail of the magnetosphere. Since it is expected that most particles entering near the equator in the tail of the magnetosphere will drift across the tail once they enter, it is reasonable to expect a cross tail current system directed from dawn to dusk with protons moving from dawn to dusk and electrons moving from dusk to dawn.

Also, since only one species of charge carrier enters on a given side of the magnetosphere, it is likely that entry will be retarded due to an accumulation of positive charge just within the dawn side magnetopause and negative charge on the dusk side of the magnetosphere. This charge accumulation is expected to occur inside of but near the magnetopause boundary on both sides of the tail of the magnetosphere. It is in the proper sense to cause a dawn to dusk

electrostatic field to persist across the bulk of the tail region. It also produces a dusk to dawn electric field of considerably larger magnitude in the "boundary layer" between the charge layer and the magnetopause on both the dawn and dusk sides of the magnetosphere, which will work together with the magnetospheric magnetic field to cause antisolar flow of plasma in the boundary layer region as is observed. Since this plasma transport is occuring near the magnetic equator in the tail, it quite naturally explains the persistence of the plasma sheet there. The fact that electrons cannot enter on the dawn side nor protons on the dusk side leads to another charge accumulation just outside of the magnetopause near the equator. We believe this accumulation of charge is dissipated as it supplies energy to drive the return circuit for the cross tail currents. A cross section of the magnetospheric tail region illustrating qualitatively the several magnetospheric features explained as a consequence of the gradient drift entry mechanism is shown in Figure 2. Although we have not yet quantitatively explored the formation of the "dayside cusps" in terms of the GDE process, it seems clear to us that the neutral point regions described by the old pressure balance formalism will give way to the observed more extended cusp like entry region. Our work on the GDE mechanism has been described in greater detail elsewhere (Olson and Pfitzer, 1983; Olson, 1983).

We may now restate our current physical understanding of the nature of the formation and maintenance of gross magnetospheric features. The pressure balance formalism, together with its assumption of specular reflection, can be used to quite accurately describe the size and shape of the magnetosphere and the topology of the currents that flow on the magnetopause. However, it is important to recognize that the assumption of specular reflection is indeed an approximation and that even for particles with solar wind energies a substantial fraction of them continuously enter the magnetosphere. This entry is permitted because in reality even over the region of the initial interaction of solar wind particles with the geomagetic field, there is enough of a gradient in the field to permit some particles to gain entry into the magnetosphere. Because of the gradient in the magnetic field only positively charged particles may enter the magnetosphere on the dawn side and only

negatively charged particles on the dusk side. This asymmetry results in the formation and maintenance of the cross tail current system, the Birkeland Region I currents, a boundary layer electric field directed from dusk to dawn on both sides of the magnetosphere and a cross tail electric field directed from dawn to dusk at all times. We also note that although the pressure balance formalism models were described as "closed", there is no such thing as a magnetosphere that is closed to the entry of charged particles, even low energy (solar wind) particles. More will be said on the open versus closed description of the geomagnetic field below. Thus, by simply extending the old pressure balance formalism to include the consequences of the observed gradients in the geomagnetic field, it is possible to explain qualitatively the existence of most large scale features that persist at all times in the magnetosphere.

1.2 Magnetospheric Response to Solar Wind Variability

As mentioned above, in addition to explaining the existence of the gross features observed in the magnetosphere, it is also necessary to understand how they respond to changes in the interplanetary magnetic field and solar wind. For example, the pressure balance formalism equates solar wind streaming pressure with the energy density of the geomagnetic field. As solar wind density and/or velocity change, the location of the magnetopause and the size of the magnetosphere change in response. Thus, it is possible to quantitatively model changes in the magnetospheric magnetic field due to changes in solar wind pressure. Likewise it should be possible to explain the variability of other magnetospheric features as they respond to changes in solar wind and interplanetary magnetic field parameters. Some work has been done on this and quantitative models have been developed for specific magnetospheric event epochs (Olson and Pfitzer, 1982). We note here briefly that the several processes and features in the magnetosphere that depend on the GDE mechanism vary roughly as the gyro radius of the incident particles in the geomagnetic field. Thus, any change in the density of impacting particles, their velocity, or in the magnitude and direction of the geomagnetic field is expected to impact several magnetospheric features.

1.3 Magnetospheric Response to the Interplanetary Magnetic Field

In addition to magnetospheric variability associated with changes in the solar wind, a large body of observational data has shown that several magnetospheric features are also controlled by the presence and variability of the interplanetary magnetic field (IPMF). The IPMF consists of a background or ambient component with period of weeks, referred to as the IPMF sector structure, and higher frequency components with periods of minutes to many hours. The ambient or sector structure field normally lies in the ecliptic plane and either points away or toward the sun at the classic "garden hose" angle. It has been the goal of our work for ONR this past year to investigate quantitatively the interaction of the IPMF with the magnetospheric magnetic field. The idea that the magnetosphere is under control of the IPMF dates back to Dungey (1961) and Levy et al. (1964). They suggested that the magnetospheric magnetic field was "open" to the interplanetary magnetic field, or "reconnected" with the interplanetary field for certain directions of the IPMF. Thus they suggested that the magnetosphere was at times open to the entry of charged particles that would flow along those field lines that were joined between the interplanetary and magnetospheric regions. Proponents of this "reconnection theory" suggest that a southward pointing IPMF is especially interesting for the magnetosphere because in addition to allowing particles to get into the magnetosphere, at the same time this field geometry produces a dawn to dusk electric field across the tail. (Our disagreement with this explanation of the cross tail electric field is discussed below.)

We have at least two major objections to the reconnection theory. First, it has only been promulgated on a qualitative basis, and second, we do not believe that the interplanetary magnetic field can produce on a steady and permanent basis the many large scale magnetospheric features that are continuously observed. We choose instead to believe that the magnetosphere is formed and maintained primarily by the interaction of the geomagnetic field with the solar wind plasma. The basic magnetospheric mechanism then responds to changes in both solar wind parameters and to the interplanetary magnetic field.

We are therefore led to examine the existence of magnetic disturbances in the interplanetary region and their interaction with the magnetosphere on a physical basis. To do this, it is first necessary to characterize the interplanetary region in terms of the plasmas and fields that persist there. We then attempt to describe quantitatively the persistence of the observed magnetic fields in the interplanetary region with emphasis on the forms of disturbances that can propagate in that medium. We then examine the interaction of these disturbances with the magnetosphere and attempt to explain on a sound physical basis the response of the magnetosphere to changes in the interplanetary magnetic field. Details of this work, including the complete mathematical development, is presented in the appendices.

- 1.4 Conclusions on Wave Propagation in the Solar Wind

 The results of this examination of the propagation of the electromagnetic disturbances in solar wind may be summarized as follows:
- 1. It is clear that the interplanetary field can persist only in the presence of the solar wind plasma. In the absence of a plasma, the presence of a time varying magnetic field in interplanetary space of the magnitude of only a few nT would have associated with it an electric field with a magnitude on the order of 1 volt per meter, which is at least three orders of magnitude larger than the magnitude of electric fields observed in the interplanetary region.
- 2. In the solar wind, in the absence of a background ambient magnetic field, any disturbances with periods on the order of minutes to hours would be rapidly attenuated unless they are driven continuously in a local region.
- 3. We are thus led to explain the propagation of interplanetary electromagnetic disturbances in the presence of both the solar wind plasma and a "steady state" magnetic field. When both of these conditions are present, electromagnetic waves with periods from a few minutes to several hours can propagate over distances large with respect to the magnetosphere size

without appreciable attenuation. The background (or ambient) magnetic field is provided by the "solar sector magnetic field" which is co-produced with the solar wind and moves outward from the sun with the solar wind, and has a period of about two weeks — much longer than the characteristic periods of the electromagnetic disturbances being considered. Electromagnetic waves allowed in the interplanetary medium will propagate with the Alfven speed. There are two wave modes that propagate without appreciable attenuation.

- (a) When the propagation vector is parallel to the ambient magnetic field.
- (b) When both the propagation vector and the disturbance electric field are perpendicular to the ambient magnetic field direction. We note that when the propagation vector of the electromagnetic wave is perpendicular to the ambient magnetic field and the electric field is parallel to the ambient magnetic field, the wave is damped as if the ambient magnetic field were not present.

1.5 Reflection and Refraction at the Magnetopause

It is recalled that our interest in understanding the propagation of electromagnetic disturbances in interplanetary space is not for its own sake, but rather to help us explain the dependence of many magnetospheric processes on the presence and variability of the interplanetary magnetic field. In order to understand how electromagnetic waves propagating in interplanetary space may influence magnetospheric processes, it is necessary to examine the reflection and refraction of electromagnetic waves at the magnetopause (a discontinuity in the plasma).

At the magnetopause, the propagation direction of electromagnetic disturbances will bend in accordance with Snell's law (Note: Snell's law is valid even for anisotropic media). Thus

k_{in} sin
$$\theta_{in} = k_{out} \sin \theta_{out}$$

where k_{in} and k_{out} are the propagation vectors inside and outside the magnetopause and θ_{in} and θ_{out} are the angles the k vector makes with the normal to the interface surface. Snell's law holds for a plane wave interacting with an infinite flat surface and can be used to good approximation over the tail of the magnetopause. In the nose and dayside cusp regions of the magnetopause region, it can be used only as a semi-quantitative indicator.

If $k_{out}/k_{in} \le 1$ (i.e., the same order or smaller), then the disturbance can enter through the interface for all angles of incidence. If, however, k_{out}/k_{in} is large, then entry can occur only near perpendicular incidence ($\theta_{out} \cong 0$).

It is shown in Appendix B that for the low frequency disturbances of interest, only Alfven-like modes can propagate and then only in the presence of a d.c. magnetic field (i.e., the disturbance must be superimposed on a steady state field). Using the definitions of the plasma frequency, ω_p , and the cyclotron frequency for ions, Ω_i , we can write

$$\frac{k_{\text{out}}}{k_{\text{in}}} = \frac{\left[\left(\omega_{\text{p}}\right)_{\text{i}}\right]_{\text{out}}}{\left[\left(\omega_{\text{p}}\right)_{\text{i}}\right]_{\text{in}}} \cdot \frac{\left(\Omega_{\text{i}}\right)_{\text{in}}}{\left(\Omega_{\text{i}}\right)_{\text{out}}}$$

$$= \left(\frac{n_{\text{out}}}{n_{\text{in}}}\right)^{1/2} \cdot \frac{\left(B_{\text{amb}}\right)_{\text{in}}}{\left(B_{\text{amb}}\right)_{\text{out}}}$$

We now estimate the ratio $k_{out}/k_{in}/(n_{out} = 5/cm^3)$ and $(B_{amb})_{out} = 2 \text{ nT}$) and various magnetospheric conditions

| ⁿ inside | ide Binside Kout ^{/k} in | | Entry | |
|---------------------|-----------------------------------|-----|----------|--|
| 1/cm ³ | 50 nT | 55 | Poor | |
| 10/cm ³ | 2 nT | .4 | Good | |
| 1/cm ³ | 2 nT | 1.4 | Okay | |
| .1/cm ³ | 2 nT | 4.4 | Marginal | |

Thus entry into a magnetospheric region of high field strength or into a very rarefied plasma region is difficult, whereas entry into a denser plasma region or into a weak field region is relatively easy. For example, the wave can penetrate the flanks of the tail with relative ease (i.e., all directions of the k vector can enter), whereas entry into the tail lobes is difficult.

Since we expect transmission of the wave through various portions of the boundary, especially the flanks of the tail, an estimate of the transmission factors must be made. The actual matching of the fields at a boundary of an anisotropic medium is quite complicated. However, for normal incidence the problem is quite straightforward since the Fresnel field ratios are the same as for isotropic media. Thus

$$\frac{E_{in}}{E_{out}} = \frac{2 k_{out}}{k_{out} + k_{in}} = \frac{2}{1 + \frac{k_{in}}{K_{out}}}$$

If
$$\frac{k_{out}}{k_{in}} \approx .5$$
 then

$$\frac{E_{in}}{E_{out}} = .67$$
 and the

associated magnetic fields are

$$\frac{B_{in}}{B_{out}} = \frac{k_{in}}{k_{out}} \cdot \frac{E_{in}}{E_{out}} \approx 2 (.67) \approx 1.3$$

Thus in this example, the magnetic field disturbance has a larger B amplitude within the magnetosphere than in interplanetary space. It is expected that generally the B_{in} will be larger than B_{out} over those regions of the magnetosphere where entry of the wave is easy (where the plasma density is high and the ambient magnetic field strength is low, e.g., the flanks of the tail).

This suggests that electromagnetic disturbances can easily penetrate into the magnetosphere near the equatorial flanks of the tail. For the disturbance wave to propagate inside the magnetosphere, its $\underline{\underline{E}}$ vector must be perpendicular to the ambient (magnetospheric magnetic) field, $\underline{\underline{B}}_{Ai}$, and its $\underline{\underline{B}}$ vector must be parallel to $\underline{\underline{B}}_{Ai}$. Since $\underline{\underline{B}}_{Ai}$ in the equatorial tail region is in the north direction, only north-south disturbances can enter and propagate through the magnetosphere. Other waves can enter but will be rapidly damped. When this constraint is coupled with the requirement that, while propagating in interplanetary space, the disturbance $\underline{\underline{E}}$ vector must be perpendicular to the ambient field, $\underline{\underline{B}}_{Ao}$, only one geometry is allowed. In it, $\underline{\underline{B}}_{Ao}$ is directed toward or away from the magnetopause (parallel or antiparallel to the propagation vector) with $\underline{\underline{B}}$ parallel (or antiparallel) to $\underline{\underline{B}}_{Ai}$. It is shown in the next paragraph that this geometry can occur frequently owing to vector structure geometry.

We now examine propagation and entry when in the basic "garden hose" direction. The sector field, \underline{B}_{AO} , lies primarily in the ecliptic plane, and points either "away" or "toward" the sun along the direction of an Archimedes spiral. Disturbances traveling parallel to the "garden hose" direction of the IPMF may have any orientation of th \underline{B} and \underline{E} disturbance vectors, whereas disturbances traveling perpendicular to \underline{B}_{AO} must have their \underline{B} disturbance vector in the ecliptic plane (the \underline{E} disturbance vector cannot be parallel to \underline{B}_{AO}). Inside the magnetosphere the \underline{B} disturbance vector must be north-south or perpendicular to the ecliptic plane. Thus only waves traveling along the IPMF (the "garden hose" angle) with their \underline{B} disturbance vector in the north-south (perpendicular to ecliptic) direction can enter equatorial magnetosphere and subsequently propagate within the magnetosphere. Waves traveling perpendicular to the IPMF will be damped after entry.

The determination of the reflection/transmission values for \underline{E} and \underline{B} for non-perpendicular entry are beyond the scope of this work. However, since Snell's law is satisfied, substantial entry is expected for disturbances arriving from non-perpendicular directions.

1.6 Technical Summary

The presence of a low frequency sector structure magnetic field permits the propagation of magnetic disturbances in the solar wind. Disturbances moving parallel to this ambient field propagate without appreciable dissipation. Disturbances moving perpendicular to the ambient solar field propagate only when the B vector of the disturbance is parallel to the ambient magnetic field.

When the disturbance field encounters the magnetopause, penetration may or may not occur depending on the plasma relationships between the magnetosphere and interplanetary space. Near the equatorial flanks of the magnetospheric tail most angles of incidence are permitted, thus entry of the disturbance field through the boundary is possible. Once inside, the k vector will be nearly perpendicular to the internal ambient magnetic field, and thus only the mode in which the B of the disturbance field is parallel to B_{amb} can propagate within the magnetosphere. The mode with B initially perpendicular to the ambient magnetospheric field will be quickly damped to zero.

This has the interesting consequence that a disturbance field entering near the equator of the tail can propagate only if the B vector after entry is parallel to the existing magnetospheric field. Thus the field along the equatorial flanks of the plasma sheet is either strengthened or weakened. If a north (south) variation in the IPMF persists, the B_Z component in the center of the plasma sheet will be strenthened (weakened). It is well known that the southward turning interplanetary field initiates substorms. This analysis shows that a southward turning field will extensively weaken the field in the plasma sheet. We propose to significantly expand our investigation in the consequences of the interplanetary field for magnetospheric behavior.

The following is a summary of important findings and conclusions reached in work performed this past year.

- Observations of interplanetary electric and magnetic fields indicate an E/B ratio that is inconsistent with wave propagation in a vacuum. We have found that the presence of the solar wind plasma exerts a profound influence on E and B in interplanetary space. Also, the common inference of a cross tail electric field associated with a southward pointing interplanetary magnetic field is based on the Lorentz transformation of electromagnetic fields in vacuum and does not apply to the solar wind-interplanetary field-magnetosphere interaction which is dominated by the presence of plasma.
- o The magnetospheric magnetic field acts as the ambient (background) field within the magnetosphere while the low frequency magnetic field associated with the rotation of the sun (the solar sector field) acts as the ambient field in the solar wind. Its presence is required for the propagation of other higher frequency electromagnetic disturbances without appreciable attenuation.
- o We find that two modes of propagation are allowed in the solar wind:
 - Propagation vector parallel to the background magnetic field
 - Propagation vector <u>and</u> electric field perpendicular to the background magnetic field.
- o At the magnetopause, such disturbances are reflected and refracted. A portion of the field can penetrate into the magnetosphere. Its properties are determined by the magnetospheric parameters (e.g., plasma density, "ambient" magnetospheric magnetic field).
- o Penetration of interplanetary magnetic disturbances occurs most readily in the equatorial region of the tail (in the plasma sheet) where the field strength is low and plasma density relatively high.

Magnetic disturbances in the north or south direction most significantly influence the tail plasma sheet region. The entry of a northward disturbance field decreases the beta (ratio of particle kinetic energy to magnetic field energy density) while a southward field increases beta. We note that for a given ambient magnetospheric field, the entry of a southward disturbance field has more effect than that of a northward disturbance of the same magnetosphere on the percent change in beta. This suggests that our work may be important for the understanding of the magnetospheric substorm process which is known to respond dramatically to a southward pointing IPMF.

We believe that much of the magnetosphere's structure and dynamics can be explained in terms of our entry theory and this physical explanation of the interaction of the IPMF with the magnetosphere. We note that this work is substantially different from the reconnection theories currently in voque, and feel that our work holds promise for understanding much of the magnetosphere's structure and dynamics on a quantitative, physical basis.

Section 2

FUTURE WORK

2.1 Oblique Incidence

In our work to date, we have treated only the algebraically simple case of perpendicular incidence of interplanetary electromagnetic disturbances on the magnetopause. In order to understand the consequences of electromagnetic disturbances as they are incident upon and enter the magnetosphere, it is necessary to understand at all points on the magnetopause which portion of an electromagnetic wave is reflected and which portion is refracted. To do this we must expand the present analysis to include the oblique incidence of electromagnetic disturbances (for non-zero angles between k and normal to the magnetopause surface). This task is algebraically demanding but well understood.

2.2 The Effects of Interplanetary Electromagnetic Disturbances on Magnetospheric Plasmas and Fields

It is expected that the incidence of the interplanetary magnetic field upon the magnetosphere and its penetration into the magnetosphere will impact several magnetic features and processes. The first feature the interplanetary disturbance encounters is the magnetopause and boundary layer. We expect that the gradient drift entry mechanism (discussed above) will itself be influenced by the interplanetary field. For example, in the equatorial magnetosphere, both northward and southward interplanetary field directions can penetrate the magnetosphere. A 2 nT interplanetary magnetic field will be amplified as it enters the magnetosphere, thus in a region where the ambient magnetospheric magnetic field may be 5 nT directed northward, the interplanetary magnetic field influence will change the total field to range between 8 and 2 nT, depending on the direction of the interplanetary field. As mentioned earlier, variations in the plasma sheet field strength will change the number of particles entering the magnetosphere via the gradient drift entry process. Within the magnetosphere, the influence of interplanetary magnetic

disturbances will depend largely on both the strength of the magnetic field and the density of the plasma in the region under consideration. Changes in total magnetic field strength will, of course, influence the drift of charged particles in that region. The propagation of the disturbance wave through the magnetosphere depends on these plasma and field properties also. For example, the propagation speed of the wave (at the Alfven velocity) is directly proportional to the strength of the magnetic field in the region and inversely proportional to the square root of the plasma density. Thus, in a region like the plasma sheet where the plasma density is relatively large and the magnetic field small, the propagation velocity of the disturbance is quite small. For the high end of observed values in the plasma sheet region, it therefore may take the disturbance a few hours to cross the magnetosphere. This is to be contrasted with the lobe regions where the Alfven velocity may be ten to a hundred times larger.

In order to understand totally the effects of these interplanetary disturbances on the magnetosphere, it is necessary not only to calculate the magnitude of the penetrating magnetic field and the velocity of the wave in the magnetosphere, but to use that information to examine the plasma dynamic processes which operate in the magnetosphere. For example, a southward pointing interplanetary magnetic field disturbance gradient drift process will move through the tail region and decrease the total field in the plasma sheet, the plasma sheet particles should react. We would also, therefore, propose to examine at least qualitatively the effects of the presence of these interplanetary disturbances in the magnetosphere on both the magnetospheric substorm phenomenon and on the classical magnetic storm signature.

2.3 Influences in the Dayside Cusp Regions

Additionally, we feel it important to examine the interaction between these interplanetary disturbances and the magnetosphere in the dayside cusp region. It is well known that the auroral and polar regions are influenced more by the toward/away changes in the sector structure magnetic field than the north/south direction changes that influence the tail region. We expect that

it is possible to understand this dependence in terms of the ways that these interplanetary disturbances (and the ambient sector structure magnetic field) interact with the magnetosphere or magnetospheric magnetic field in the region of the dayside cusps. This task would also initially be approached in a qualitative manner since its rigorous solution will require not only an understanding of oblique incidence of disturbances, but also the relaxation of our other assumption used thus far—that we can represent the disturbance as a plane wave. Clearly in the dayside cusp region there is enough structure to the topology of the magnetospheric magnetic field that a plane wave approximation for the disturbance is questionable. We believe that we can examine this problem qualitatively and offer an explanation with a physical basis for the coupling between the interplanetary magnetic field and several auroral and polar cap phenomena.

1

Appendix A

GENERAL DESCRIPTION OF SOLAR WIND AND INTERPLANETARY MAGNETIC FIELD PROPERTIES

Interplanetary space is characterized by the continuous presence of both charged particles and a magnetic field. The most persistent feature of the interplanetary medium is the solar wind which flows approximately radially outward from the sun. The solar wind is typically characterized in terms of its bulk speed, the thermal energy of both ions and electrons, and its density. It is essentially electrically neutral. Its bulk speed has been observed to range from under 300 to 1,000 kilometers per second. The thermal energy of the solar wind protons is approximately 10 eV corresponding to a thermal velocity of approximately 5 x 10^4 m/sec. The electron bulk speed is approximately the same as that of the protons, but the electron thermal velocity is approximately 2 x 10^6 m/sec. The solar wind density is much more variable than its bulk speed, ranging from less than .1 to over 50 particle pairs per cubic centimeter (10^5 to 5 x 10^7 per 3^6).

The continuous flow of the solar wind is frequently interrupted by the passage of more energetic plasmas which also originate from the sun. There are many classes of interplanetary disturbances. All of them have shorter scale lengths and characteristic periods than the steady solar wind. We can therefore characterize the interplanetary plasma basically in terms of solar wind parameters if we understand that these average solar wind parameters are frequently perturbed by the passage of other plasmas.

Interplanetary space is also characterized by the presence of a magnetic field, commonly referred to as the interplanetary magnetic field (IPMF). The strength of the interplanetary field characteristically ranges from a large fraction of a nT to 25 nT. Typically its strength ranges from 2 to 5 nT. The interplanetary magnetic field, as will be shown below, is carried with the solar wind and is also perturbed by the passage of energetic plasmas. Like the interplanetary plasmas, the interplanetary magnetic field may be described

as an average field (like the solar wind) perturbed by other fluctuating fields. The average field is produced by the 28 day rotation of the sun. It is referred to as the solar sector structure of the interplanetary magnetic field and results, on the average, in four distinct regions or sectors per 28 day rotation, in which the magnetic field direction is primarily directed either away or toward the sun. These sectors are referred to as toward and away sectors. This background portion of the interplanetary magnetic field has a period of approximately 2 weeks and may, for all purposes, be considered a constant field when contrasted with the periodicities typically associated with the other magnetic disturbances which persist in interplanetary space. For example, changes in the IPMF associated with magnetospheric substorms and magnetic storms, typically last on the order of a few hours. Other changes in the IPMF of interest to our studies typically persist anywhere from a few minutes to several hours, and range in magnitude from about 1/10 nT to several nT.

In order to understand the interaction of the interplanetary magnetic field with the magnetosphere, we have first examined the propagation of electromagnetic disturbances in the interplanetary medium. Prior to the determination of allowed propagation modes, it is appropriate to examine some characteristic frequencies and other parameters in the interplanetary medium.

The approximate plasma frequencies for solar wind electrons and protons are given by

$$(\omega_{\mathbf{p}})_{\mathbf{j}}^{2} = \frac{q_{\mathbf{j}}^{2} n_{\mathbf{j}}}{m_{\mathbf{j}}^{2} c_{\mathbf{0}}} \tag{1}$$

where $(\omega_p)_j$ is the plasma frequency for the jth species, n_j , q_j , and m_j are the number density, charge, and mass for the jth species. The plasma frequency for protons $(\omega_p)_i$ and electrons $(\omega_p)_e$ is then found to be in the range

$$(\omega_{\rm p})_{\dot{1}} \approx 4 \times 10^2$$
 to $10^4/{\rm sec}$ (2) $(\omega_{\rm p})_{\dot{e}} \approx 2 \times 10^4$ to $4 \times 10^5/{\rm sec}$

The cyclotron frequency is given by

$$\Omega_{j} = \frac{q_{j} B}{m_{j}}$$
 (3)

Thus if the magnetic field, B, is in the range 1 to 50 nT, the cyclotron frequencies are:

$$\Omega_{\rm i} \approx 0.1$$
 to 5/sec for protons (4)
$$-\Omega_{\rm e} \approx 200$$
 to 10 /sec for electrons

The Debye shielding length, λ_D , for a plasma is given by

$$\lambda_{n} = k_{n}^{-1} \tag{5}$$

where

$$k_0 = \frac{(\omega_p)_e}{\sqrt{\overline{v}_e^2}} + \frac{(\omega_p)_i}{\sqrt{\overline{v}_i^2}}$$

where \overline{v}_e and \overline{v}_i are the r.m.s. thermal speed of the electrons and protons. We note that the electron and ion species contributions to k_D are approximately equal. Thus typically in the solar wind the Debye length is

 $\lambda_0 \approx 2.5$ to 50 meters

Therefore, within a Debye sphere $n_p \lambda_D^3$ and $n_e \lambda_D^3 >> 1$. It is therefore appropriate to treat the solar wind as a plasma and to use plasma collective mode equations to describe electromagnetic processes present there.

Also, since the highest frequency disturbances do not excede 10^{-2} hz, their wavelengths far exceed the Debye length λ_D and the long wavelength approximation can be used.

Another plasma parameter of importance in this analysis is the collision frequency. The dominant collision mechanism is the close-in collision between the charged particles due to the Coulomb force. This Coulomb scattering (Rutherford) is discussed in many texts (for example, see Jackson, 1962 and Davies, 1966).

The momentum transfer collision frequency of the electrons due to interactions with the protons (ions), $v_{\rm el}$, is

$$v_{ej} = n_j \sigma_R \tilde{V} (1 - \cos \theta)$$
 (6)

where n_i is the ion density, σ_R is the Rutherford cross section, \tilde{V} is the average speed between particles (approximately the r.m.s. electron thermal speed of 2 x 10⁶ m/sec) and $(\overline{1-\cos\theta})$ is the mean change in the direction cosine of the electrons caused by the proton. Rutherford scattering theory gives

$$(1-\cos\theta) \simeq \frac{\overline{\theta^2}}{2} = \theta_{\min}^2 \ln \left(\frac{\theta_{\max}}{\theta_{\min}}\right)$$
 (7)

where $\theta_{\mbox{min}}$ is the minimum scattering angle and $\theta_{\mbox{max}}$ is the maximum scattering angle. The ratio

$$\frac{\theta_{\text{max}}}{\theta_{\text{min}}} = \frac{12 \cdot \sqrt{n_i}}{k_0^3} \tag{8}$$

For a screened plasma of temperature k_B T \leq Rydberg (13.7 eV), where k_B is the Boltzman constant and T is the temperature, the Rutherford cross section is

$$\sigma_{r} = \begin{vmatrix} \frac{2 \ \frac{7}{e} \ \frac{7}{i} \ \frac{e^{2}}{\rho} \end{vmatrix} = \frac{\pi}{\frac{2}{e_{\min}}}$$
(9)

where

 $Z_e = Z_{i} = 1$ the charge of the two species

$$p = M_r v$$
, and

 $1/M_r = 1/m_i + 1/m_e$ the reduced mass $M_r = m_e$ because $m_i >> m_e$; therefore, by combining equations (20)-(23), one has

$$v_{ei} = \frac{4\pi n_i e^4}{M_r^2 \tilde{v}^3} \text{ in } (\frac{12 \pi n_i}{k_n^3})$$
 (10)

Since $k_0 = 2(\omega_p)_e/\tilde{v}$ and for the case of $n_i = 5/cc$ (Note: Eq. (10) is in unrationalized c.g.s. units), one gets $v_{ei} = 5 \times 10^{-7}/sec$.

The momentum transfer collision for ions due to collision with electrons is a factor $m_e/M_i \simeq 1/2000$ smaller than v_{ei} , therefore $v_{ie} = 2(10^{-10})$ sec⁻¹ for the above specified plasma density.

The electron-electron collision frequency v_{ee} is given by a similar analysis with a reduced mass $M_r = m_e/2$. Thus $v_{ee} = 2 \ (10^{-6})$ sec⁻¹. The total collision frequency for electrons. v_e , is given by

$$v_e = v_{ei} + v_{ee} = 2.5 \times 10^{-6}/\text{sec}$$

The ion-ion collision frequency, v_{ij} , is given by a similar formula, but since $M_r = m_i/2 >> m_e$, v_{ij} is negligible compared to v_{ie} . Therefore the total collision frequency for ions, v_i , is

$$v_i \approx v_{ie} = 2 \times 10^{-10}/\text{sec}$$

Since we are interested in disturbances with periods from minutes to weeks, the disturbance frequency ω ranges from 0.1 sec⁻¹ to 10^{-6} sec⁻¹. We note that in our examination of the propagation of electromagnetic waves in the solar wind, the plasma parameters have the following properties:

$$v_i < v_e \lesssim \omega < \Omega_i < -\Omega_e$$

The relations between these frequencies are important to the analysis that follows.

Appendix B

PROPAGATION OF ELECTROMAGNETIC DISTURBANCES IN THE SOLAR WIND

We now proceed to examine the characteristics of electromagnetic disturbances that form in the solar wind and determine the properties of those disturbance modes that can propagate large distances without appreciable attenuation.

B.1 Propagation in a Vacuum

It is instructive first to examine wave propagation in a vacuum. In a true vacuum, the ratio E to B is (E/Bc=1). Therefore, in a vacuum, a 2 nT disturbance has associated with it an electric field of ~ 0.6 volts/meter. Thus it is obvious that we cannot use the vacuum approximation for magnetospheric work since the observed electric fields are smaller by at least three orders of magnitude.

We also note that the relativistic transformation for electromagnetic fields are typically stated for vacuum conditions. Therefore, it is incorrect to assume that the presence of the interplanetary field as viewed from an earth reference frame produces in that frame (moving with velocity V with respect to the solar wind) an electric field. The question of interplanetary electric fields and their properties in the magnetosphere is in reality made much more complicated by the presence of plasma in the interplanetary region. Thus the dawn to dusk cross tail electric field that has been inferred to persist during periods of southward pointing interplanetary magnetic field is not only a gross oversimplification, but basically incorrect.

B.2 Propagation in the Presence of a Plasma

B.2.1 General Equations

To treat the general case of electromagnetic waves in the solar wind, it is first noted that the frequency range of disturbances observed in the IPMF is many orders of magnitude lower than the frequency of wave phenomena typically studied in the laboratory. Yet, we will find that magnetic disturbances

present in the solar wind are not magnetostatic phenomena but electromagnetic waves. To represent electromagnetic waves in the presence of a plasma, it is customary to begin with Maxwell's equations.

If the plasma has disturbances at the angular frequency, ω (sec⁻¹), then the electric field, \underline{E} , and magnetic field, \underline{B} , can be written

$$\underline{\mathbf{E}} = \underline{\mathbf{E}}_{0} e^{i \omega t}$$

$$\underline{\mathbf{B}} = \underline{\mathbf{B}}_{0} e^{i \omega t}$$
(11)

where \underline{B}_0 and \underline{E}_0 are dependent only on position and t is the time.

Maxwell's equations in rationalized MKS units in frequency space can be written

$$\nabla \times \underline{E} = -i\omega \underline{B}$$

$$\nabla \times \underline{B} = \mu_0 \underline{J} + \frac{i\omega}{c^2} \underline{E}$$

$$\nabla \cdot \underline{B} = 0$$

$$\nabla \cdot \underline{E} = \frac{\rho}{c_0}$$
(12)

Where

$$\mu_0 = 4\pi (10^{-7}) \text{ henries/m}$$
 $\epsilon_0 = 8.85 (10^{-12}) \text{ farads/m}$
 $c = 3(10^8) \text{ m/s}$

and ρ and \underline{J} are the total charge and current densities respectively.

It is convenient to break the charge and current sources (ρ and \underline{J}) into externally driven sources (ρ_{ext} and \underline{J}_{ext}) and sources produced locally (ρ_{ind} and \underline{J}_{ind}) by polarization effects.

Thus

$$\rho = \rho_{\text{ext}} + \rho_{\text{ind}} = \rho_{\text{ext}} - \nabla \cdot (\epsilon_{\text{o}} \times \underline{\epsilon})$$
(13)

 $\underline{J} = \underline{J}_{ext} + \underline{J}_{ind} = \underline{J}_{ext} + \underline{g} \cdot \underline{E}$

where \underline{X} and $\underline{\sigma}$ are the tensor functions for susceptibility and conductivity.

The curl of the Equation (7) yields

$$\nabla \times \nabla \times \underline{E} = -i \omega \nabla \times \underline{B}$$

$$(14)$$

$$\nabla(\nabla \cdot \underline{E}) - \nabla \cdot \nabla \underline{E} = -i \omega \nabla \times \underline{B}$$

Since

$$\nabla \times \underline{\beta} = \mu_0 \underline{J} + \frac{i \omega}{c^2} \underline{E}$$
 and
$$\nabla \cdot \underline{E} = \rho/\epsilon_0$$

it follows that

$$\nabla(\frac{\varrho}{\varepsilon_0}) - \nabla^2 \underline{E} = -i \omega \left(\mu_0 \underline{J} + i \frac{\omega}{c^2} \underline{E}\right)$$
 (15)

Substituting for ρ and \underline{J} by using Equation (8),

$$- \nabla^{2} \underline{E} - \frac{\omega^{2}}{c^{2}} \underline{E} - \nabla(\nabla \cdot \underline{X} \cdot \underline{E}) + i \mu_{0} \omega \underline{g} \cdot \underline{E} =$$

$$- i \omega \mu_{0} \underline{J}_{ext} - \nabla \left(\frac{\rho_{ext}}{\epsilon_{0}}\right)$$
(16)

Note that Equation (11) is the frequency space-domain, and

$$\underline{E} = \underline{E}(\omega, \underline{r}) = \underline{E}(\underline{r}) e^{i\omega t}$$
.

The Fourier transform of the space domain into vector wave numbers, \underline{k} , permits the substitution of a differential Equation (11) by an algebraic equation (replacing ∇ with $i\underline{k}$). Thus

$$(k^{2} - \frac{\omega^{2}}{c^{2}}) \stackrel{\underline{E}}{=} + \underline{k}(\underline{k} \cdot X \cdot \underline{E}) + i \mu_{0} \omega \stackrel{\underline{g} \cdot \underline{E}}{=}$$

$$= -i(k \frac{\rho_{ext}}{\epsilon_{0}} + \mu_{0} \omega \stackrel{\underline{J}}{=} ext)$$
(17)

The right side of Equation (17) represents the transform of the various source terms creating the disturbance. When these source terms are known, the algebraic equation can be solved. Then, by inverting the transform, the space time dependence of the fields is obtained.

We seek field patterns that can exist (i.e., frequency and wave number relations) without being continually excited (i.e., that can propagate in the solar wind plasma). They are obtained by setting the right side of Equation (17) to zero. For a non-zero solution of the resulting homogeneous equation, the determinant of the coefficients must vanish such that

$$\det \left\{ \left(k^2 - \frac{\omega^2}{c^2} \right) \, \underline{1} + \underline{k} (\underline{k} \cdot \underline{X}) + i \, \mu_0 \, \omega \, \underline{\sigma} \, \right\} = 0 \tag{18}$$

To solve Equation (18), it is necessary to develop a relationship between the tensor electrical conductivity $\underline{\sigma}$ and magnetic susceptibility, $\underline{\chi}$. Since the solar wind is known to be almost perfectly charge neutral at all times, conservation of charge can be used to provide the required relation. Thus

$$\nabla \cdot \underline{J}_{ind} + \frac{\partial \rho_{ind}}{\partial t} = 0$$
 (19)

then by substituting for \underline{J}_{ind} and ρ_{ind} , we get

$$\nabla \cdot (\underline{\sigma} - \mathbf{1} \ \omega \ \epsilon_0 \underline{X}) \cdot \underline{E} = 0 \tag{20}$$

As mentioned earlier, for the frequencies of interest (of magnetic disturbances in the solar wind) the long wavelength approximation can be used. Thus

$$g = i \omega \epsilon_0 \chi$$
 (21)

Using Equation (16), Equation (13) can be written in terms of only \underline{x} .

$$\det \{ (k^2 - \frac{\omega^2}{c^2}) \ \underline{1} + \underline{k}(\underline{k} \cdot \underline{x}) - \frac{\omega^2}{c^2} \ \underline{x} \ \} = 0$$
 (22)

To arrive at the solution to Equation (22) and provide the appropriate relation between k and ω , we must first arrive at a solution to the susceptibility tensor X.

The cold plasma and long wavelength characteristics are most easily obtained from the Lorentz force equation with collisional damping (used by Appelton in his ionospheric work of the 1920's).

$$m_{j} \frac{d\underline{V}_{i}}{dt} = q_{j} (\underline{E} + \underline{V}_{j} \times \underline{B}_{amb}) - v_{j} m_{j} \underline{V}_{j}$$
 (23)

where

 \mathbf{m}_{i} is the mass of the jth particle

 \underline{v}_{j} is the frequency transform of the velocity vector of the j^{th} particle species

q_i is the charge

 v_j is the collisional frequency damping of the j^{th} particle species

B_{amb} is the ambient magnetic field

In this work only two particle species are important, protons and electrons. If we let the subscript j = e for electrons and j = i for protons (ions), then

$$m_e = 9.1 (10^{-31}) \text{ kg}$$
 $m_i = 1.6 (10^{-27}) \text{ kg}$
 $q_e = +e = 1.6 \times 10^{-19} \text{ coulomb}$
 $q_i = -e = -1.6 \times 10^{-19} \text{ coulomb}$

Furthermore, since $(\underline{v_j} \cdot \nabla \underline{v_j}) \ \underline{v_j} << av_j/at$ in the long wavelength approximation, then

$$\frac{d \underline{v}_{i}}{dt} \approx \frac{\partial \underline{v}_{i}}{\partial t}$$

Equation (23) can be rewritten as

$$i \omega \underline{V}_{j} = \frac{q_{j}}{m_{j}} (\underline{E} + \underline{V}_{j} \times \underline{B}_{amb}) - v_{j} \underline{V}_{j}$$
 (24)

This above vector equation is a set of coupled linear equations and can be solved for \underline{V}_j in a straightforward manner. To solve this set of equations, a right handed Cartesian coordinate system is defined with unit vectors $\underline{\alpha}$, $\underline{\beta}$, $\underline{\gamma}$, where $\underline{\gamma}$ is the direction of the ambient magnetic field \underline{B}_{amb} and $\underline{\gamma} = \underline{\alpha} \times \underline{\beta}$. To simplify the algebra, we define the cyclotron frequency of the j^{th} species, Ω_j , as $\Omega_j = q_j \frac{B_{amb}}{m_j}$. Solving Eq. (24), one gets

$$(V_{j})_{\alpha} = -\frac{i \frac{q_{j}}{m_{j}} (\omega - i v_{j})}{(\omega - i v_{j})^{2} - \Omega_{j}^{2}} E_{\alpha} - \frac{\frac{q_{j}}{m_{j}} \Omega_{j}}{(\omega - i v_{j})^{2} - \Omega_{j}^{2}} E_{\beta}$$

$$(V_{j})_{\beta} = -\frac{i \frac{q_{j}}{m_{j}} (\omega - i v_{j})}{(\omega - i v_{j})^{2} - \Omega_{j}^{2}} E_{\beta} + \frac{\frac{q_{j}}{m_{j}} \Omega_{j}}{(\omega - i v_{j})^{2} - \Omega_{j}^{2}} E_{\alpha} \qquad (25)$$

$$(v_j)_{\gamma} = -\frac{i \frac{q_j}{m_j}}{(\omega - i v_j)} E_{\gamma}$$

Since

$$(\underline{J}_{ind})_{j} = q_{j} n_{j} \underline{V}_{j}$$
 (26)

and since $(\underline{J}_{ind})_j$ may also be written as

where n_j is the density of the j^{th} species, then

$$q_{j} n_{j} \underline{V}_{j} = \underline{g}_{j} \cdot \underline{E}$$
 (28)

Substitution in equation (6) gives

$$\frac{q_{j} n_{j}}{i \omega \epsilon_{0}} \underline{V}_{j} = \underline{x}_{j} \cdot \underline{E}$$
 (29)

Thus one sees that except for the multiplicative constant, $q_j n_j/i \omega \epsilon_0$, Equations (9) and (13) are the same. Thus the complex electric susceptibility tensors may be written down.

$$(X_{j})_{\alpha\alpha} = (X_{j})_{\beta\beta} = \frac{-(\omega_{p})_{j}^{2} (\omega - i v_{j})}{\omega [(\omega - i v_{j})^{2} - \Omega_{j}]^{2}}$$

$$(X_{j})_{\gamma\gamma} = \frac{-(\omega_{p})_{j}^{2}}{\omega (\omega - i v_{j})}$$

$$(X_{j})_{\alpha\beta} = (X_{j})_{\beta\alpha} = \frac{i(\omega_{p})_{j}^{2} \Omega_{j}}{\omega [(\omega - i v_{j})^{2} - \Omega_{j}^{2}]}$$

$$(X_{j})_{\alpha\gamma} = (X_{j})_{\gamma\alpha} = (X_{j})_{\beta\gamma} = (X_{j})_{\gamma\beta} = 0$$

$$(30)$$

where

$$(\omega_p)_j^2 = \frac{q_j^2 n_j}{m_j \epsilon_0}$$
 and $\Omega_j = q_j \frac{B}{m_j}$

 $(\omega_p)_{j\, th}$ is the plasma frequency and Ω_j is the cyclotron frequency for the j^{th} species.

The values of the susceptibility tensor are now substituted into equation (22) and expanding this equation into component form one gets

$$\begin{vmatrix} A + k_{\alpha}m, & C + k_{\alpha}n, & k_{\alpha}q \\ -C + k_{\beta}m, & A + k_{\beta}n, & k_{\beta}q \\ k_{\gamma}m, & k_{\gamma}n, & B + k_{\gamma}q \end{vmatrix} = 0$$

where

$$A = k^{2} - \frac{\omega^{2}}{c^{2}} (1 + X_{\alpha\alpha}),$$

$$B = k^{2} - \frac{\omega^{2}}{c^{2}} (1 + X_{\gamma\gamma}),$$

$$C = -\frac{\omega^{2}}{c^{2}} X_{\alpha\beta}$$

$$m = (k \cdot X)_{\alpha} = k_{\alpha} X_{\alpha\alpha} + k_{\beta} X_{\beta\alpha} = k_{\alpha} X_{\alpha\alpha} - k_{\beta} X_{\alpha\beta}$$

$$n = (k \cdot X)_{\beta} = k_{\alpha} X_{\alpha\beta} + k_{\beta} X_{\beta\beta} = k_{\alpha} X_{\alpha\beta} + k_{\beta} X_{\alpha\alpha}$$

$$q = (k \cdot X)_{\gamma} = k_{\gamma} X_{\gamma\gamma}$$

$$(31)$$

(m, n, q are the projections of the \underline{x} tensor on the \underline{k} vector and k_{α} , k_{β} , and k_{γ} are the components of the \underline{k} vector in the $\underline{\alpha}$, $\underline{\beta}$ and $\underline{\gamma}$ direction.)

Expanding the determinant gives

$$A^{2}B + AB(k_{\alpha}m + k_{\beta}n) + A^{2}k_{\gamma}q + BC^{2} + BC(k_{\alpha}n - k_{\beta}m) + C^{2}k_{\gamma}q = 0$$
 (32)

If we define the transverse component of the <u>k</u> vector, k_T , as $\underline{k}_T = k_{\alpha}$ $\underline{a} + k_{\beta}\beta$, thus

$$k_{T}^{2} = k_{\alpha}^{2} + k_{\beta}^{2} \tag{33}$$

then

$$k_{\alpha}m + k_{\beta}n = k_{T}^{2} X_{\alpha\alpha}$$

$$k_{\alpha}n - k_{\beta}m = k_{T}^{2} X_{\alpha\beta}$$

$$k_{\gamma}q = k_{\gamma}^{2} X_{\gamma\gamma}$$
(34)

$$\{ [k^{2} - \frac{\omega^{2}}{c^{2}} (1 + X_{\alpha\alpha})]^{2} + [\frac{\omega^{2}}{c^{2}} X_{\alpha\beta}]^{2} \} \{ k^{2} - \frac{\omega^{2}}{c^{2}} (1 + X_{\gamma\gamma}) + k_{\gamma}^{2} X_{\gamma\gamma} \}$$

$$+ k_{T}^{2} [k^{2} - \frac{\omega^{2}}{c^{2}} (1 + X_{\gamma\gamma})] [(k^{2} - \frac{\omega^{2}}{c^{2}}) X_{\alpha\alpha} - \frac{\omega^{2}}{c^{2}} (X_{\alpha\alpha}^{2} + X_{\alpha\beta}^{2})]$$

$$= 0$$

$$(35)$$

B.2.2 Propagation in a Plasma with No Background Magnetic Field

We now apply the above set of equations to the magnetospheric disturbance problems. We first examine the propagation of a disturbance (a wave of frequency ω) through the solar wind when no "steady" magnetic field is present. When B = 0, the cyclotron frequencies for both electrons and ions are zero ($\Omega_{\bf i}=\Omega_{\bf e}=0$). Setting $\Omega_{\bf i}$ and $\Omega_{\bf e}$ to zero and solving Equation (35) gives

$$X_{\alpha\beta} = X_{\beta\alpha} = 0$$

$$(X_{j})_{\alpha\alpha} = (X_{j})_{\beta\beta} = (X_{j})_{\gamma\gamma}$$
(36)

If we let

$$X = (X_1)_{\alpha\alpha} + (X_e)_{\alpha\alpha} = (X_1)_{\beta\beta} + (X_e)_{\beta\beta} = \dots$$

then

$$X = -\frac{(\omega_{\mathbf{p}})_{1}^{2}}{\omega(\omega - i \nu_{1})} - \frac{(\omega_{\mathbf{p}})_{\mathbf{e}}^{2}}{\omega(\omega - i \nu_{\mathbf{e}})}$$
(37)

If we take Equation (35) and apply the conditions in (36), we get

$$[k^{2} - \frac{\omega^{2}}{c^{2}} (1 + X)]^{2} [k^{2} - \frac{\omega^{2}}{c^{2}} (1 + X) + k_{\gamma}^{2} X]$$

$$+ k_{T} [k^{2} - \frac{\omega^{2}}{c^{2}} (1 + X)]^{2} [(k^{2} - \frac{\omega^{2}}{c^{2}}) X - \frac{\omega^{2}}{c^{2}} X^{2}] = 0$$
(38)

This equation can be satisfied if

$$k^2 - \frac{\omega^2}{c^2} (1 + X) = 0 (39)$$

We can substitute Equation (37) into (38) and since $\omega_p >> \omega$ and |X| >> 1, one gets

$$k^{2} = \frac{\omega^{2}}{c^{2}} \left[-\frac{\left(\omega_{p}\right)_{i}^{2}}{\omega(\omega-i v_{i})} - \frac{\left(\omega_{p}\right)_{e}^{2}}{\omega(\omega-i v_{e})} \right]$$
(40)

Furthermore, since $(\omega_p)_e >> (\omega_p)_i$, we can drop the first term and thus we can write

$$k^{2} \approx \frac{-(\omega_{p})_{e}^{2}}{c^{2}} \frac{\omega^{2}}{(\omega^{2} + \nu_{e}^{2})} - i \frac{(\omega_{p})_{e}^{2}}{c^{2}} \frac{\omega \nu_{e}}{(\omega^{2} + \nu_{e}^{2})}$$
(41)

Since the real part of the above equation is always negative, there is no real solution for k. Furthermore, since ω is on the same order or larger than v_e , the magnitude of the complex k is $\sim (\omega_p)_e/c$, which is on the order of 10^{-4} to 10^{-3} /meter. Over lengths of 1 to 10 km, any electromagnetic wave will be Debye shielded. Thus we find that electromagnetic waves cannot propagate in the solar wind plasma when it does not possess a steady background magnetic field unless they are continuously driven by local sources. In other words, any electromagnetic disturbance formed in the solar wind source will die out over a scale length of 1 to 10 km if no background magnetic field is present.

B.2.3 Propagation in a Plasma With an Imbedded Magnetic Field

As discussed in the introduction, it is recalled that magnetic disturbances in the solar wind can be separated roughly into two categories; high frequency perturbations and the very low frequency disturbances associated with the 28 day rotation of the sun. The frequency associated with the 28 day period magnetic field is so much lower than the range of disturbance frequencies of interest that it may be considered for our purposes to provide a background magnetic field to the solar wind.

Generally, in the presence of both a plasma and an imbedded steady state magnetic field, the ratio of E/Bc = $\omega/\text{kc} = \omega/\omega_{\text{p}}$. Since $\omega << \omega_{\text{p}}$, the electric field associated with magnetic variations the solar wind is very small. For the 28 day solar rotation source, the resultant electric field of $\sim 10^{-9}$ volt/meter is much smaller than observed. Thus the observed interplanetary electric field ($\sim 10^{-6}$ to 10^{-4} volt/meter) must be produced by the higher frequency disturbances (on the order of hours or minutes) in which we are interested.

We now show that these higher frequency disturbances are allowed to propagate in the solar wind when it contains a background magnetic field associated with the sun's rotation (the solar sector magnetic field).

We have shown quite generally that <u>for a low frequency wave to propagate</u> without excessive damping and with the proper E/B relationship, the presence of a plasma and a background steady magnetic field are both required.

To examine the progation of electromagnetic disturbances in the solar wind (containing "ambient" solar sector magnetic field), it is convenient to examine two distinct cases: 1) propagation along the ambient magnetic field, and 2) propagation normal to the ambient magnetic field.

B.2.3.1 Propagation Parallel to B

For propagation along the ambient field

$$k_T = 0$$

$$k^2 = k_{\Upsilon}^2$$

Thus the dispersion relation, Equation (35) can be rewritten as

$$\{[k_{\Upsilon}^{2} - \frac{\omega^{2}}{c^{2}}(1 + X_{\alpha\alpha})]^{2} + (\frac{\omega^{2}}{c^{2}}X_{\alpha\beta})^{2}\} (1 + X_{\Upsilon\Upsilon})(k_{\Upsilon}^{2} - \frac{\omega^{2}}{c^{2}}) = 0$$
 (42)

Here, the first factor is zero if

$$k_{\Upsilon}^{2} = \frac{\omega^{2}}{c^{2}} (1 + \chi_{\alpha\alpha} \pm i \chi_{\alpha\beta})$$

Using the values for x_i and x_e in Equation (18), and substituting for $x_{\alpha\beta}$ and $x_{\alpha\beta}$ yields

$$X_{\alpha\alpha} \pm iX_{\alpha\beta} = -\frac{(\omega_{p})_{i}^{2} (\omega - iv_{i})}{\omega[(\omega - iv_{i})^{2} - \Omega_{i}^{2}]} - \frac{(\omega_{p})_{e}^{2} (\omega - iv_{e})}{\omega[(\omega - iv_{e})^{2} - \Omega_{e}^{2}]}$$

$$\pm \frac{i \cdot i (\omega_{p})_{i}^{2} \Omega_{i}}{\omega[(\omega - iv_{i})^{2} - \Omega_{i}^{2}]^{\pm}} - \frac{i \cdot i (\omega_{p})^{2} \Omega_{e}}{\omega[(\omega - iv_{e})^{2} - \Omega_{e}^{2}]}$$
(43)

Simplifying gives

$$X_{\alpha\alpha} \pm iX_{\alpha\beta} = -\frac{-(\omega_{p})_{i}^{2}}{\omega(\omega - in_{i} + \Omega_{i})} - \frac{(\omega_{p})_{e}^{2}}{\omega(\omega - in_{e} + \Omega_{e})}$$

$$= \frac{-(\omega_{p})_{i}^{2}}{-(\omega_{p})_{i}^{2}} - \frac{(\omega_{p})_{e}^{2}}{-(\omega_{p})_{e}^{2}}$$

Since ν_1 and ω << Ω_1 , and ν_e and ω << Ω_e

and since
$$\frac{1}{1 + \xi} = 1 + \xi + 0(\xi^2)$$
 for $\xi << 1$

and since
$$\frac{(\omega_p)_1^2}{\Omega_1} = \frac{-(\omega_p)_e^2}{\Omega e^-}$$
 when $ne = n_1$

then

$$X_{\alpha\alpha} \pm i X_{\alpha\beta} \approx \frac{(\omega_{p})_{i}^{2}}{\Omega_{i}^{2}} \left(\frac{\omega - i v_{i}}{\omega}\right) + \frac{(\omega_{p})_{i}^{2}}{\Omega_{i}^{2}} \left(\frac{\omega - i v_{e}}{\omega}\right) \tag{45}$$

utilizing the facts

$$\Omega_{\rm e} - = \frac{\rm m_{\rm i}}{\rm m_{\rm e}} \, \Omega_{\rm i}$$

$$\frac{m_1}{m_0} >> 1$$

one then gets

$$X_{\alpha\alpha} \pm i X_{\alpha\beta} \approx \frac{(\omega_{p})_{i}^{2}}{\Omega_{i}^{2}} \left[\frac{\omega - i (v_{i} + v_{e} \frac{m_{e}}{m_{i}})}{\omega} \right]$$
 (46)

Substituting into equation (43) gives

$$k_{\gamma} = + \frac{\omega}{c} \left[1 + \frac{(\omega_{p})_{i}^{2}}{\Omega_{i}^{2}} \left(1 - \frac{i \left(v_{i} + v_{e} \frac{m_{e}}{m_{i}} \right)}{\omega} \right) \right]^{1/2}$$
 (47)

In our regime of interest

$$\frac{v_i}{\omega} \ll 1 \quad , \quad \frac{v_e \frac{m_e}{m_i}}{\omega} \ll 1$$

$$\frac{(\omega_p)_i^2}{\Omega_i^2} \gg 1$$

$$\frac{(\omega_{\rm p})_{\rm i}^2}{\Omega_{\rm s}^2} >> 1$$

Therefore

$$k_{\gamma} = \pm \frac{\omega}{c} \frac{(\omega_{p})_{i}}{\Omega_{i}} \left[1 - \frac{i \left(v_{i} + v_{e} \frac{m_{e}}{m_{i}}\right)}{\omega}\right]^{1/2}$$
(48)

$$\approx \pm \frac{\omega}{c} \frac{(\omega_{p})_{i}}{\Omega_{i}} \left[1 - \frac{i(v_{i} + v_{e} \frac{m_{e}}{m_{i}})}{2\omega}\right]^{1/2}$$

$$= \pm \frac{\omega \left(\omega_{\mathbf{p}}\right)_{i}}{c \Omega_{i}} + \frac{i \left(v_{i} + v_{e} \frac{m_{e}}{m_{i}}\right)(\omega_{\mathbf{p}})_{i}}{2 c \Omega_{i}} = \text{Real}\left(k_{\gamma}\right) + i \text{ Imag}\left(k_{\gamma}\right)$$
 (49)

Real
$$(k_{\gamma}) = \pm \frac{\omega}{c} \frac{(\omega_{p})_{\dot{1}}}{\Omega_{\dot{1}}}$$
 (50)

Imag
$$(k_{\gamma}) = \frac{-\frac{(v_i + v_e \frac{m_e}{m_i})(\omega_p)_i}{2 c \Omega_i}$$

This describes a wave having a phase velocity $\approx \frac{c \, \Omega_i}{(\omega_p)_i}$ (which is the same size as the well known Alfven velocity) that is much slower than c. Since for the typical solar wind and IPMF values,

$$\frac{\Omega_{i}}{(\omega_{n})_{i}} = 5 \times 10^{-4} \text{ to } 2.5 \times 10^{-3}$$

the phase velocity $V_A \approx \frac{\omega}{k}$ ranges from 1.5 x 10^{-5} to 7.5 x 10^5 meters/sec.

Since E \approx B V_A, a 2 nT disturbance has associated with it a 0.3 to 1.5 millivolt/meter electrostatic field.

Since the fields vary as $e^{-|[Imag (k)]|r||}$, the distance over which the wave decreases to 1/e is given by

$$\xi = \frac{1}{|\text{Imag}(k_{\gamma})|} = \frac{2 c \Omega_{i}}{(v_{i} + v_{e} \frac{m_{e}}{m_{i}})(\omega_{p})_{i}}$$
(51)

For typical solar wind values ($n_i = 5/cc = 5 (10^6)$ m/sec, $B_{amb} = 2$ nT, $(\omega_p)_i = 3 \times 10^{+3}/\text{sec}$, $\Omega_i = .5/\text{sec}$, $v_e = 2.5 \times 10^{-6}/\text{sec}$, $v_i = 2 \times 10^{-10}/\text{sec}$), $\xi = 6.9 \times 10^{13}$ meter = 6.9 x 10^{10} km. This implies that disturbances in the interplantary region propagating in the direction of the ambient magnetic field (the solar sector structure magnetic field) are very weakly damped and essentially can propagate without significant retardation over distances large with respect to the scale size of the magnetosphere. In fact, there is little attenuation even over distances on the order of an a.u. (the distance between the earth and the sun). It is observed that several disturbances in the interplanetary region persist over distances up to a considerable fraction of an astronomical unit. Also, the solar sector structure clearly persists over several a.u. It is thus necessary to show that a portion of the interplanetary magnetic field is "born" with the solar wind and the field and plasma then move together away from the sun, each in a sense controlling the other. Our work on this topic will be described elsewhere.

We summarize this section on wave propagation in the direction of the ambient magnetic field by stating that such propagation can occur without significant attenuation and that such disturbances propagate approximately at the Alfven velocity.

B.2.3.2 Propagation Perpendicular to B

In the normal propagation mode

$$k^2 = k_T^2 = k_{\alpha}^2 + k_{\beta}^2$$

Setting $k_{\gamma} = 0$ in Equation (19) and rearranging terms yields

$$[k_{T}^{2} - \frac{\omega^{2}}{c^{2}}(1 + X_{YY})][k_{T}^{2}(1 + X_{\alpha\alpha}) - \frac{\omega^{2}}{c^{2}}(1 + 2X_{\alpha\alpha} + X_{\alpha\alpha}^{2} + X_{\alpha\beta}^{2})][k_{T}^{2} - \frac{\omega^{2}}{c^{2}}] = 0 \quad (24)$$

The first factor $[k_T^2 - \omega^2/c \ (1 + \chi_{\gamma\gamma})]$ has the same form as in the nonambient magnetic field configuration, and thus represents the same highly damped mode. This equation only involves $\chi_{\gamma\gamma}$ and expanding the tensor in Equation (17), it is seen that $\chi_{\gamma\gamma}$ affects only the E term. Thus a wave propagating perpendicular to μ_{amb} having its E vector parallel to the ambient magnetic field is highly damped.

A similar expansion of the tensor in Equation (17) shows that the term

$$[k_{T}^{2} (1 + X_{\alpha\alpha}) - \frac{\omega^{2}}{c^{2}} (1 + 2 X_{\alpha\alpha} + X_{\alpha\beta}^{2} + X_{\alpha\beta}^{2})]$$
 (53)

represents a wave with both its propagation vector and its electric field vector oriented perpendicular to the direction of the ambient magnetic field.

To solve this mode we must solve

$$k_{T}^{2} (1 + X_{\alpha\alpha}) - \frac{\omega^{2}}{c^{2}} (1 + 2 X_{\alpha\alpha} + X_{\alpha\alpha}^{2} + X_{\alpha\beta}^{2}) = 0$$
 (54)

This gives

$$k_{T} = \pm \frac{\omega}{C} \sqrt{\frac{1+2 + \frac{\chi_{\alpha\alpha} + \chi_{\alpha\beta}^{2} + \chi_{\alpha\beta}}{1+\chi_{\alpha\alpha}}}{1+\chi_{\alpha\alpha}}}$$

$$= \pm \frac{\omega}{C} \sqrt{\frac{(1+\chi_{\alpha\alpha} + i + \chi_{\alpha\beta})(1+\chi_{\alpha\alpha} - i + \chi_{\alpha\beta})}{1+\chi_{\alpha\alpha}}}$$
(55)

From Equations (43) and (44) it is obvious that

$$1 + X_{\alpha\alpha} + iX_{\alpha\beta} = 1 + X_{\alpha\alpha} - iX_{\alpha\beta}$$

$$\approx \left[1 + \frac{(\omega_p)_2^2}{\Omega_i^2} \left(1 - \frac{i(\nu_i + \nu_e \frac{m_e}{m_i})}{\omega}\right)\right]^{1/2}$$
 (56)

Using the same magnitude approximations used in simplifying Equation (45), one can show that

$$1 + X_{\alpha\alpha} = \frac{\left(\omega_{\rm p}\right)_{\rm f}^2}{\Omega_{\rm f}^2} \tag{57}$$

$$k_{T} = \frac{\omega}{c} \frac{\frac{(1 + X_{\alpha\alpha} + i X_{\alpha\beta})}{(1 + X_{\alpha\alpha})^{1/2}}}{\frac{i(v_{i} + v_{e} \frac{m_{e}}{m_{i}})}{2\omega}}$$

$$= \frac{\omega}{c} \frac{(\omega_{p})_{i}}{\Omega_{i}} \left[1 - \frac{i(v_{i} + v_{e} \frac{m_{e}}{m_{i}})}{2\omega}\right]$$
(58)

This suggests that a wave propagating perpendicular to the background magnetic field and with its electric field also perpendicular to the ambient magnetic field will travel in the same mode as the case considered above for propagation parallel to the direction of the ambient magnetic field. Again, the wave will propagate with a velocity close to the Alfven speed.

REFERENCES AND RECENT PUBLICATIONS

REFERENCES

Davies, K. and Appleton, <u>Ionospheric Radio Propagation</u>, National Bureau of Standards Monograph <u>80</u>, pp 63-99.

Dungey, J. W. Interplanetary Magnetic Field and the Auroral Zones, Phys. Rev. Lett., 6, 47, 1961.

Jackson, J. D. Classical Electrodynamics, Chapter 13, pps 429-463, J. Wiley & Son, New York (1962).

Levy, R. H., H. E. Petscheck, and G. L. Siscoe, Aerodynamical Aspects of the Magnetospheric Flow, <u>AIAA J.</u>, 2, 2065, 1964.

Olson, W. P., and K. A. Pfitzer, A Dynamic Model of the Magnetospheric and Electric Fields for July 29, 1977, <u>J. Geophys. Res</u>., 87, A8, 5943-5948, 1982.

Olson, W. P. and K. A. Pfitzer. The Entry of Magnetosheath Plasma as a Source of Steady State Magnetospheric Features. Submitted to the $\underline{\text{J. Geophys. Res.}}$, May 1983.

Olson, W. P. Introduction to the Topology of Magnetospheric Current Systems. Geophysical Monograph Series, Vol. 28, (T. A. Potemra, Editor). Published by The American Geophysical Union, Washington, D.C., January 1984.

RECENT PERTINENT PUBLICATIONS

Olson, W. P. Introduction to the Topology of Magnetospheric Current Systems. Geophysical Monograph Series, Vol. 28, (T. A. Potemra, Editor). Published by The American Geophysical Union, Washington, D.C., January 1984.

Olson, W. P. and K. A. Pfitzer. The Entry of AMPTE Lithium Ions into a Magnetically Closed Magnetosphere. <u>J. Geophys. Res.</u>, 89, 7337, 1984.

Olson, W. P. and K. A. Pfitzer. The Entry of Magnetosheath Plasma as a Source of Steady State Magnetospheric Features. Submitted to the $\underline{\text{J. Geophys. Res.}}$, May 1983.